Numerical modelization by finite differences of a thermoelectric refrigeration device of “double jump". Experimental validation.

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Abstract

We have been developed a mathematical model to study the operation of a thermoelectric refrigerator device with double jump. The computational model solves both thermoelectricity and heat transfer equations. The model calculates the value of the internal temperature and the COP of the refrigerator.

In an experimental phase a prototype was built to adjust and validate the computational model, and later to optimize the experimental application.

The model allows simulating properly the system in both transient and steady states by using the finite differences numerical method.

We have studied the influence of the dissipater of the Peltier module in both hot and cold side, and we have made a comparison between a double jump and a single jump device.

Nomenclature

A	Area	(m^2)
C	Thermal capacity	($J\ K^{-1}$)
COP	Coefficient of performance	
k	Thermal conductivity	($W\ m^{-1}\ K^{-1}$)
l	Length	(m)
$\dot{Q}$	Heat flux	(W)
$\overset{*}{q}$	Rate of internal generation of heat	($W\ m^{-3}$)
R	Thermal resistance	($K\ W^{-1}$)
T	Temperature	(ºC)
$T\,'$	Temperature in next step	(ºC)
V	Voltage	(V)

Greek letters

ΔT	Temperature gap	(K)
τ	Time	(s)

Superscripts, subscripts

dissip	Dissipater
Exp	Experimental
i	Node i
ins	Insulation
J	Joule effect
j	Node j
Mod	Model

Introduction

Most of the applications were Peltier modules are used are cooling applications. These devices allow warming or keeping constant the temperature of objects where the variation of the temperature is important.

The thermoelectric systems are used in applications where the power range is from mW to hundreds of W as shown in references [1] and [2]. Some examples are the military industry, aerospace applications, food cooling, laboratory equipment and medical equipment. Some of these applications are considered in references [3], [4] and [5]. The thermoelectric technology offers more reliability reducing the pollution.

The case of study is the calculation of the different temperatures as a function of the time in a thermoelectric refrigerator. We will use a configuration in cascade of the cooling system, which might be formed by 3 Peltier modules. Two of these modules will be in parallel and the other in series.

In the numerical model created the user might fix the next variables:

- Room temperature
- Voltage supplied to the Peltier modules
- Time step in the numerical method used.
- The simulation time duration.

Our model allows using different numerical methods to calculate the temperatures. The user will receive the errors for each method, so they can be compared and check the convergence order, as well our study allows calculating the transient state of a double jump refrigerator before reaching the steady state.

Objectives

The principal objectives are the following:

- Development of a computational model to simulates the operation of a double jump thermoelectric refrigerator.
- Design and construction of a prototype of a double jump thermoelectric refrigerator
- Validation of the computational model developed previously
- Study of the influence of the hot and cold side dissipater in the internal temperature of the refrigerator and the COP.
- Make a study of the double jump refrigerator and single jump refrigerator.

Computational model

The basic principle of the numerical approximation to the heat transfer problems is the substitution of the differential equation for the continuous temperature in a heat conductor solid by a finite difference equation that has to be fulfilling of each points of the solid.

If a complex geometry is subdivided in a nodal net, it has to be fulfilling by the implicit formulation the following expression in each node i:

$$\sum_j \frac{t'_j - t'_i}{R_{ij}} + \dot{q}_i = \frac{C_i}{\delta\tau}\left(t'_i - t_i\right) \qquad (1)$$

The term $\dot{q}$ i, rate of internal generation of heat by the node. Thus, the node i future temperature is determined using expression (2):

$$t'_i = t_i + \delta\tau\left(\sum_j \frac{t'_j - t'_i}{R_{ij}C_i} + \frac{\dot{q}_i}{C_i}\right) \qquad (2)$$

Where the expression of the thermal resistances between nodes i and j are:

$$R_{ij} = \frac{l_{ij}}{kA_{ij}} \qquad (3)$$

If al the resistances of the net are known and the capacities of all the cells involved, thus, with a distribution of the temperature at the starting timing, equations like (2) can be written for all the nodes. The result is that several equations for future temperatures can be obtained t_i' of the nodal points as a function of the future temperatures of the nodes around, once a time step is chosen δτ.

In order to obtain the thermoelectric refrigerator COP there are several analytical expressions as shown in reference [6], where is needed to know the temperatures of the Peltier module as boundary conditions. Mathematical models (such as the developed in [7] or in this paper) calculate the COP of the thermoelectric refrigerator without knowing the temperatures of the module faces as an input parameter.

In Fig. 1 is shown the electric thermal analogy of the problem to solve. The nodes of the analogy represent the elements of the thermoelectric refrigerator.

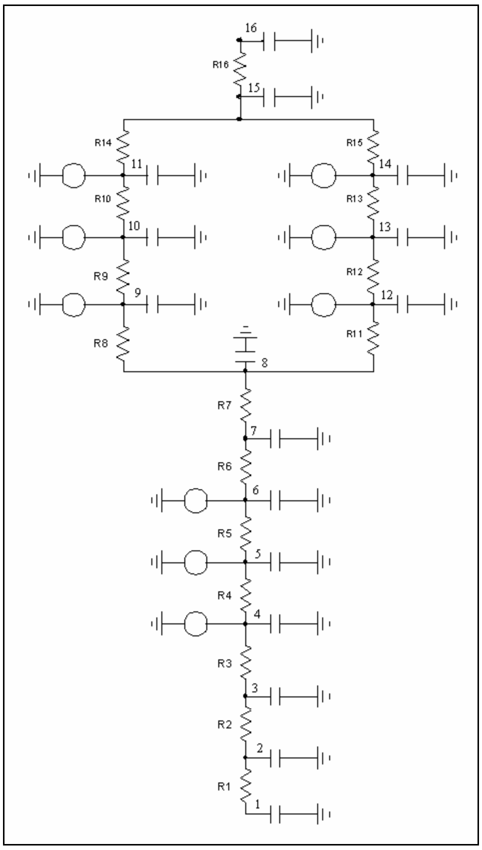

Fig. 1. double jump electric thermal analogy for a double jump refrigerator.

Each of the component temperature is associated to a node, so in the analogy showed in Fig. 1, the temperature nodes are the following:

- Node 1: Room temperature
- Node 2: Inner refrigerator temperature
- Node 3: Dissipater temperature of the Peltier module cold side.
- Nodes 4, 9 y 12: Cold side temperatures of the Peltier module
- Nodes 5, 10 y 13: Inner temperatures of the Peltier module
- Nodes 6, 11 y 14: Hot side temperatures of the Peltier module
- Node 7: Heat-extender temperature placed next to Peltier module 1.
- Node 8: Heat-extender temperature placed next to Peltier module 2 and 3.
- Node 15: Dissipater temperature of the Peltier module hot side.
- Node 16: Room temperature

With the previous discretization the following matrix system can be written:

$$[M]\cdot\left[T'_i\right] = \left[T_i\right] + \frac{\delta\tau}{C_i}\cdot\left[\dot{Q}_i\right] \qquad (4)$$

The terms $\dot{Q}_4$, $\dot{Q}_9$, $\dot{Q}_{12}$ represent the heat fluxes due to the Peltier effect in the cold side of the modules, faced to the interior of the refrigerator. The Joule effect heat fluxes are $\dot{Q}_5$, $\dot{Q}_{10}$, $\dot{Q}_{13}$ and the Peltier effects in the hot side of the modules are $\dot{Q}_6$, $\dot{Q}_{11}$, $\dot{Q}_{14}$.

The methods useds to solve the problem have been Euler, Heun and Runge-Kutta (first order methods) and Runge-Kutta 4 order method.

Experimental assembly

we have designed and built a double jump thermoelectric cooling prototype. The refrigerator used to install the thermoelectric device has a inner volume of 55*10-3 m3. The thermoelectric refrigerator system is formed by:

- A heat dissipater for the hot side of the Peltier module placed in parallel (exterior to the refrigerator)
- A heat extender that connects the more internal Peltier to the more external Peltier modules in parallel.
- A heat dissipater for the cold side introduced where the heat have to be removed.
- A third Peltier module in contact with the cold dissipater.
- A fan for the hot side dissipater.

As a scheme in Fig. 2 can be a section of the refrigerator we have developed:

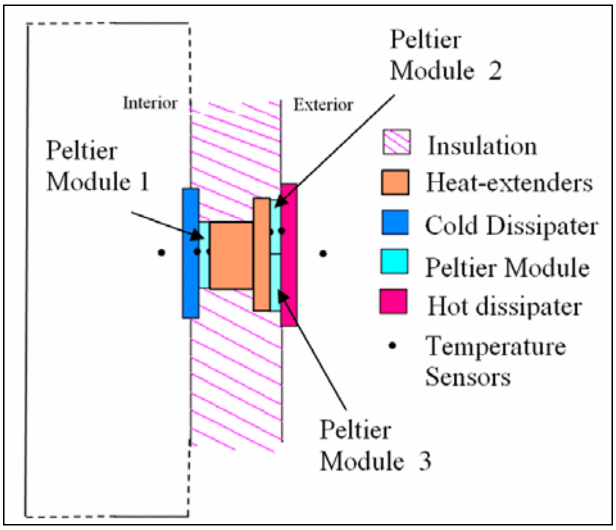

Fig. 2 Scheme of the cooling system.

The dissipaters, both hot and cold side were built from an aluminum slab in the UPNA workshop. Fins were placed in the hot side dissipater to increase the interchange surface and thus obtain a better heat dissipation.

In the cold side dissipater no fins were placed, in order to make the device lighter and not to lose useful space in the interior of the tank.

The dimensions of the dissipaters and fins are:

Hot dissipater: 400 x 155 x 15 mm

Cold dissipater: 375 x 155 x 15 mm

Fins: 400 x 50 x 1.5 mm

Once the refrigeration operation was checked and in order to improve its features, a wind tunnel was incorporated in the hot side dissipater with a fan to make forced convection on the fins of the dissipater.

The fan is Axial, (dimensions 120 x 120 x 30 mm), with a power consumption of 5W .

Model validation

In order to validate the mathematical model, the voltage of the Peltier modules and the room temperature could be changed. In Table 1 all the configurations that were tested are listed for each test, in the validation process.

	Test 1	**Test 2**	**Test 3**	**Test 4**
Peltier voltage nº 1	5 V	6 V	8 V	6 V
Peltier voltage nº 2	5 V	6 V	5 V	4 V
Peltier voltage nº 3	5 V	6 V	5 V	4 V
Fan voltage	12 V	12 V	12 V	12 V
T_{Room}	273 K	273 K	303 K	287 K

Table 1. Tests made in order to validate the thermoelectric refrigerator model.

The validation tests were made in a climatic test room in order to work with a room temperature fixed and controlled depending on the conditions of each test.

After making the tests in the laboratory with the prototype some simulations were made using the computational model. As shown in Fig. 3 the model is adjusted to the experimental values with high accuracy for both transient and steady states.

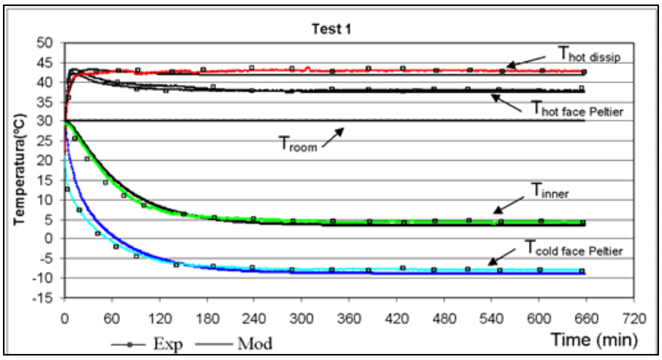

Fig. 3. Comparison between prototype and model results.

The greater difference between the model and experimental temperatures is 1.5 ºC what makes accuracy very acceptable, as we can see in the Table 2.

	Test1		Test 2		Test 3		Test 4	
	Mod	Exp	Mod	Exp	Mod	Exp	Mod	Exp
$T_{Hot\ Peltier}$ (ºC)	37.4	37.7	-4.2	-4.3	-0.7	-0.5	-4.2	-4.3
$T_{Cold\ Peltier}$ (ºC)	-8.7	-8.0	-28.3	-29.7	-31	-29.6	-28.3	-29.7
T_{Inner} (ºC)	3.4	4.3	-18.8	-20.4	-21.1	-19.5	-18.8	-20.4
$T_{Hot\ dissipater}$ (ºC)	41.7	42.6	9.9	9.7	12.9	13.5	9.9	9.7

Table 2. Comparison of the temperatures in the steady state.

Results and discussion

One of the system variables with higher influence in the thermoelectric refrigeration is the thermal resistance of the dissipaters placed for the Peltier modules, as can be seen in references [8] and [9]. The model relates the thermal resistance of the dissipaters with the simulated values of internal temperature of the refrigerator what allow us to study the influence in the refrigerator operation.

Influence of the hot side dissipater

In this point the refrigerator operation will be study as a function of the thermal resistance of the hot side dissipater. The simulated values with the matematical model were calculated modifying the datum of the thermal resistance of the hot side dissipater and keeping constant all the variables of the system. The characteristics of the simulation are the following:

- Supplied voltage of 4 V for all the Peltier modules
- Fan supplied with 12 V
- Room temperature of 0 ºC, 10 ºC and 20 ºC.

With these values a range of thermal resistance for the hot side dissipater from 0.1 K/W to 1.0 K/W were made.

The data of the temperature gap between the interior of the refrigerator and the room temperature follow a lineal trend as shown in Fig. 4. The room temperature that makes the greater thermal gap was 20 ºC.

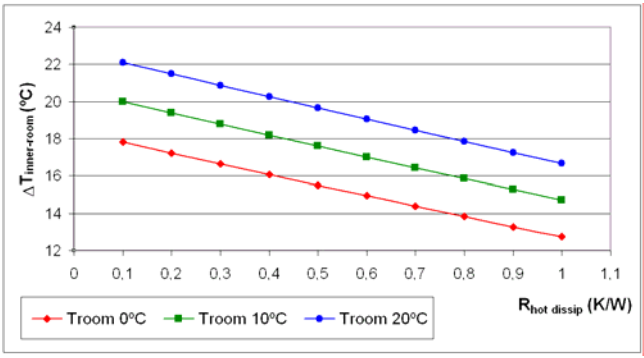

Fig.4. Thermal gap between the interior temperature and the room temperature as a function of the thermal resistance of the hot side dissipater.

As can be checked in Fig. 4, the greatest temperature gap is for a room temperature of 20 ºC, with a thermal resistance for the hot side dissipater of 0.1 K/W. In Fig. 5 are shown the results of the COP from the simulations and It can be seen that the COP's curve as a function of the hot side dissipater has the same slope for all the room temperatures studied.

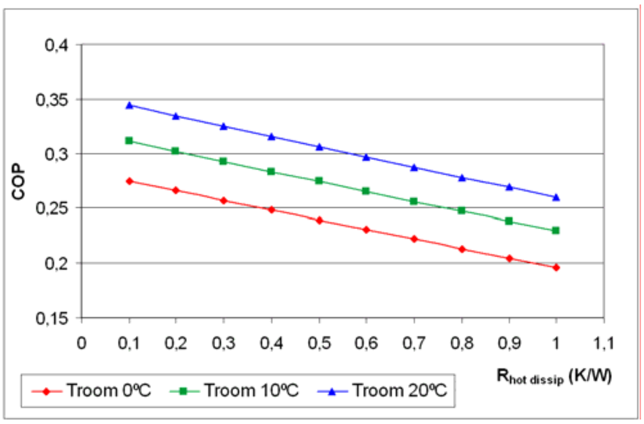

Fig. 5. COP's as a function of the thermal resistance of the hot side dissipater.

The simulations allowed us to determine that a decrease of the 10% in the thermal resistance of the hot side dissipater means an increase of the 3.4% for the COP value.

Influence of the cold side dissipater.

The simulation characteristics for the supplied Peltier voltage and fan voltage, and the test conditions were the same than in the case of the hot side dissipater. The thermal resistance value range was from 0.5 K/W to 1.5 K/W.

Checking the results from Fig. 6 it can be seen that the data present a trend similar to the hot side study.

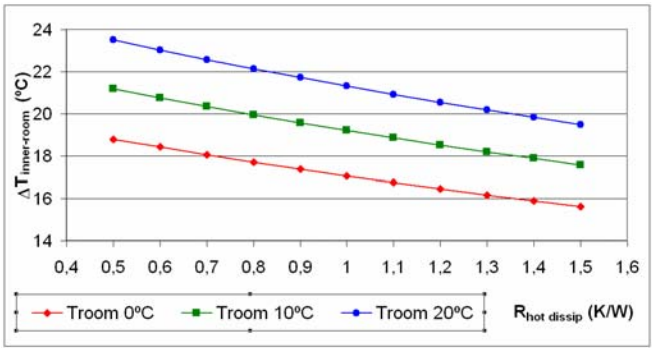

Fig. 6. Temperature gap between the internal temperature and the room temperature as a function of the thermal resistance for the cold side dissipater.

The results of the COP of the thermoelectric refrigerator are represented in Fig. 7. The greatest COP is obtained for a room temperature of 20 ºC, with the lowest thermal resistance of the cold side 0.5K/W. The COP of a thermoelectric device is a function of the room temperature where the Peltier modules are working, being necessary to use the most efficient modules for each room temperature.

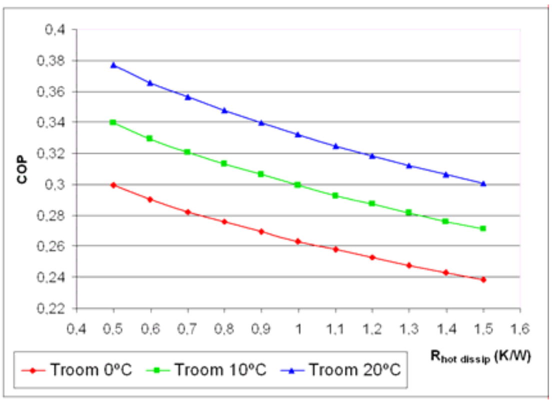

Fig.7. COP's as a function of the thermal resistance for the hot side dissipater.

A decrease of the 10% in the thermal resistance of the cold side dissipater means an increase of the 2.2% in the COP value.

The hot dissipater has a greater influence in the COP, as in a 10% of the decrease of the hot side thermal resistance means an increase of the 3,4% while in the cold side makes a 2.2% increase.

Comparison between the double jump and the single jump refrigerator.

In order to compare the operation between the double jump refrigerator (two temperature gaps due to two Peltier modules placed in series thermally) and the single jump (a single temperature gap due to the Peltier modules), the gap between the inner temperature and external room temperature will be analyzed using the model. The supplied voltages will be in a range from 1 to 12 V with a variation of 1 V between simulations.

The characteristics of the simulations are the followings:

- Simulation time: 20 h.
- Room temperature: 10 ºC.
- Cold side resistance: 1.03K/W
- Hot side resistance: 0.22 K/W

Double jump thermoelectric refrigerator.

For the double jump case, there are represented graphically the values of the temperature gap between the room temperature and the internal temperature, Fig. 8. Our model allows to calculate the voltage where the greatest temperature gap is placed. In this case, the maximum is produced while feeding the Peltier modules with 7V, getting a temperature gap of 22.7 ºC.

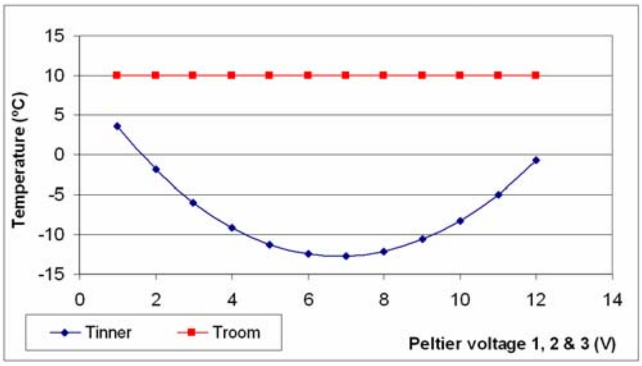

Fig. 8. Temperature gap between the interior and the exterior as a function of the voltage supplied to the Peltier modules.

It is easy to check that if the objective is to obtain the greatest temperature gap, the voltage needed is 7 V, but the COP values should be studied in order to check if that voltage produces a good COP value, or in the other hand, the power consumption is so high that another configuration with lower power consumption is recommended. The COP values are shown in Fig. 9. The best COP value is provided by a voltage of 7 V what coincides with the best temperature gap between the interior and the exterior.

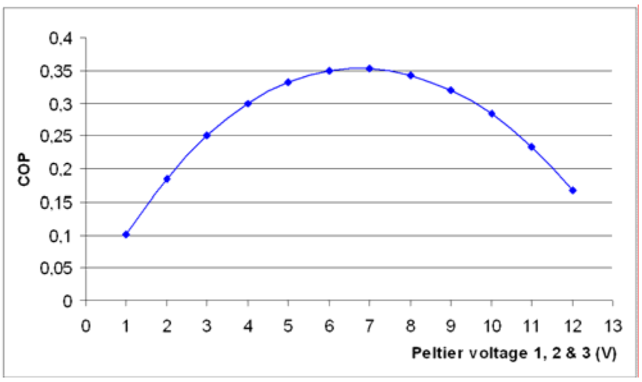

Fig. 9. COP's as a function of the voltage for a double jump refrigerator.

Single jump refrigerator.

The single jump is simulated with a 0V supplied voltage for Peltier module number 1 what makes a temperature gap between Peltier modules number 2 and 3. The optimum supplied voltage referred to the temperature gap and the COP value will be studied.

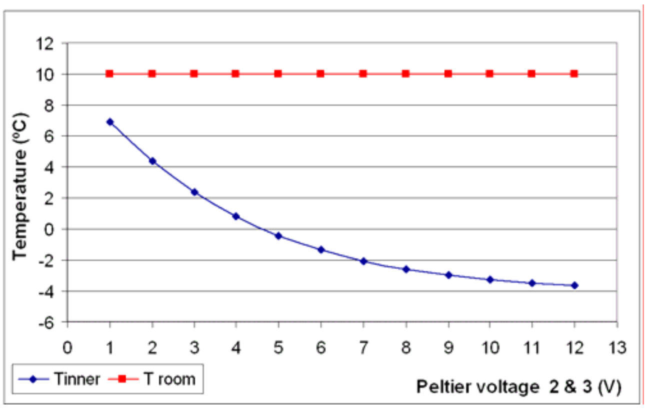

Fig. 10. COP's as a function of the voltage for the double jump refrigerator.

In this case the temperature gap is bigger as greater is the voltage of the Peltier modules, Fig 10. The slope of the curve decreases as the voltage increases. If increasing the voltage from 1 to 2 V makes a temperature gap of 25 ºC increasing the voltage from 11 to 12 V makes a temperature gap of 2 tenths of ºC.

In Fig. 11 the COP of the single jump refrigerator is shown as a function of the supplied voltage to the Peltier modules. The optimum value of COP is for a voltage of 2V for the Peltier modules, Fig. 11. In this case the maximum jump and the best value for the COP do not match at the same voltage as it happens for the double jump case. In this situation it is needed to reach a compromise between the maximum temperature gap and the best COP value.

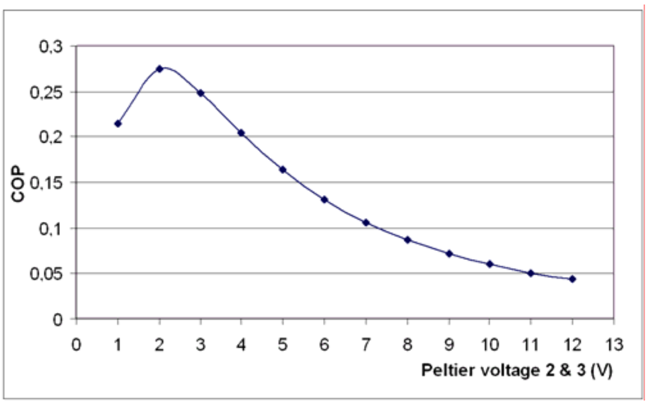

Fig. 11. COP as a function of the voltage for the single jump refrigerator.

Conclusions

The conclusions due to this work are:

- A computational model has been developed to simulate the steady and transient states for a thermoelectric double jump refrigerator, using the finite differences method. This method simulates the temperatures of each part of the thermoelectric device (nodes).
- The accuracy of the model has been checked comparing experimental and simulated values with errors lower than a 1ºC.
- The influence of the optimization of the heat dissipaters has been studied for the thermoelectric refrigerators.
- From both dissipaters, the hot side dissipater has more influence than the cold side dissipater, as a decrease of the 10% in the thermal resistance in both devices makes an increase in the COP of the 3.4% and 2.2% respectively.
- The COP for the double jump configuration is a 25% greater than the single jump configuration (when Peltier module number 1 is disconnected). Our model calculates the voltage that provides the maximum value for the COP, 7V in case of the double jump configuration and 2V for the single jump configuration.

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