Curl Currents Occurrence in Homogeneous Isotropic Thermoelectric Elements

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Abstract

Occurrence of curl currents in thermoelectric elements has been always regarded as an extremely undesirable phenomenon resulting in an inevitable efficiency reduction of the thermoelectric material. Curl currents usually appear due to local inhomogeneities in a thermoelectric material. In the given paper the occurrence of curl currents in a homogeneous material is investigated.

It is shown that in an element of the arbitrary shape the parallelism of the temperature and electric potential gradients is required to eliminate curl currents. As the electric and temperature fields in an element are described by different equations, in a general case there are no reasons for these gradients to be collinear. It means that an element of arbitrary shape always works worse than a quasi one-dimensional one, where electric and temperature fields depend on one coordinate.

We prove that for quasi one-dimensional elements in conditions of thermal exchange with environment there inevitably appear curl currents in a vicinity of the element side surface. As a result, even at the zero contact resistance there are additional reasons for the cooling efficiency of the thermoelectric module to become lower than that of the thermoelectric material it is made of.

Introduction

The most frequent inhomogeneity in semiconductors is a statistical inhomogeneity of the dopant concentration resulting in the Seebeck coefficient variation in the material. It explains the great attention usually paid to the study of the Seebeck coefficient uniformity in thermoelectric materials. In the given paper, without confining ourselves to any specific model, we investigate the reason of curl currents occurrence in a statistically homogeneous material.

Thermal processes in a thermoelectric element (or a pellet) are described by the equation [1]:

$$\nabla\vec{q} = \frac{\vec{j}^{\,2}}{\sigma} + \alpha\left(\vec{j},\nabla T\right), \tag{1}$$

where $\vec{q}$ is the energy flux density:

$$\vec{q} = -\kappa\nabla T + \alpha T\vec{j}\ , \tag{2}$$

where $\vec{j}$ is electric current density, α and σ are the Seebeck coefficient and electrical conductivity, respectively, κ is thermal conductivity of the material, and T is temperature.

The electric current inside the pellet is continuous, therefore

$$\nabla\vec{j} = 0\ , \tag{3}$$

and the relation between the electric current and electric potential is set by the equation

$$\vec{j} = -\sigma\left(\nabla\tilde{\phi} + \alpha\nabla T\right), \tag{4}$$

where $\tilde{\phi} = \phi - \frac{\mu}{e}$ is the position of the chemical potential level in the external electric field per electron.

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The boundary conditions for the standard problem of finding the electric current and temperature distribution in a thermoelectric element of any shape (Fig. 1) are as described below.

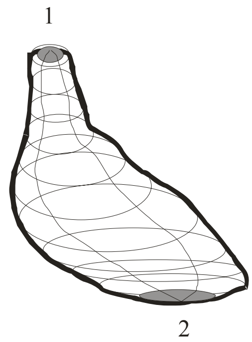

Figure 1. A schematic image of an arbitrary form element; 1 and 2 are the electrical and thermal contact surfaces.

The contact surfaces 1 and 2 (Fig. 1) are kept isothermal with temperatures T_1 and T_2. The potentials $\tilde{\phi}$ of each contact surface 1 and 2 are constant. Let $\vec{n}$ be a normal towards the lateral surface of the element. We assume that this surface is adiabatically isolated so that the normal component of the thermal flux and electric current on its side surface equals: $q_n = \left(\vec{q},\vec{n}\right) = 0$ and $j_n = \left(\vec{j},\vec{n}\right) = 0$. It results in Eqs. (2), (4) requiring that on the side surface of the element:

$$\left(\vec{n},\nabla T\right) = 0\ ,\ \left(\vec{n},\nabla\tilde{\phi}\right) = 0\ . \tag{5}$$

We assume that the thermoelectric material is isotropic and homogeneous, therefore α and σ are only temperature functions and temperature is determined by the coordinate $\vec{r}$, in which the temperature is measured. Eq. (4) easily yields:

$$\left[\nabla,\vec{j}\right] = \frac{d\sigma}{dT}\left[\nabla T\cdot\nabla\tilde{\phi}\right]. \tag{6}$$

Thus the curl currents appear when electrical conductivity depends on temperature and the directions of gradients of temperature and of electrical potential in the element do not coincide, i.e. isothermal and equipotential surfaces are different. The first requirement in the real material is always true. As for the second requirement: as the electric and temperature fields are described by different equations (1) and (3), generally there are no reasons for collinearity of vectors $\nabla\tilde{\phi}$ and ∇T in the element of the arbitrary form. Even on the lateral surface of the element where they are perpendicular to the normal (5) they lay tangentially to the lateral surface and their collinearity is not necessary at all. I.e. in a thermoelectric element of arbitrary form the curl currents always appear. Removing them requires choosing a special form of the element.

If geometry of the thermoelectric element provides for the temperature and electric fields to depend only on one spatial coordinate ξ, then curl currents are absent as Eqs. (1) and (3) become one-dimensional.

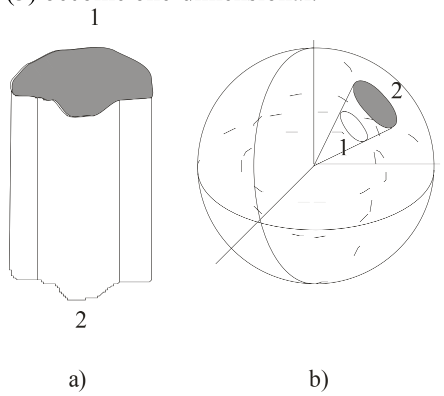

Figure 2. Shapes of thermoelectric elements in which temperature and electric field depend on one spatial coordinate: in a) they depend on distance from the contact surface; and in b) on radius. Contact surfaces in case a) are planes and in case b) are parts of spherical concentric surfaces.

As an example, we can suggest a cylinder in the adiabatic environment (Fig. 2a) or an element cut out of the hollow sphere by a radius-vector, also in the adiabatic environment. We will name such shapes quasi-one-dimensional. In case a) isothermal and isopotential surfaces are the planes parallel to the contact junctions, and in case b) they are spherical concentric surfaces. The gradients of temperature and potential in both cases are collinear and directed in case a) parallel to the cylinder axis, and in case b) along the radius-vector.

The known rule that the maximal achievable temperature difference on the ends of the thermoelectric element with surfaces being adiabatically isolated does not depend on the shape of the pellet [2] should be replaced with a stronger statement: the maximal achievable temperature difference on the ends of the arbitrarily shaped element with adiabatic isolation of walls cannot exceed the temperature difference on the ends of the quasi-one-dimensional element.

Let us assume that in a pellet of any form in a point r_1 on the pellet surface vectors $\nabla\tilde{\phi}$ and ∇T are parallel and equal $\nabla\tilde{\phi}_1$ and ∇T_1 respectively. From Eq. (4) it follows that in this case the vector of electric current density is collinear to the temperature gradient. Let us consider now this point in conditions of heat exchange with environment when the heat flux of density $q(r_1)$ falls on the lateral surface in point r_1. In this case the boundary conditions for the heat fluxes on the lateral surfaces become:

$$q_n(\vec{r}_1) = F(T_a - T(\vec{r}_1)) = -\kappa(\vec{n},\nabla T) = -\kappa\frac{\partial T}{\partial n}, \quad (7)$$

where F is the coefficient of heat exchange with environment and T_a is ambient temperature. Occurrence of the temperature gradient normal component causes the occurrence of the normal component of electric current due to the charge carriers diffusion. This current will be compensated by the change of electric potential (the Seebeck voltage) so that the aggregate normal component of the current would be equal zero. From Eq. (4) it follows, that

$$\frac{\partial\tilde{\phi}}{\partial n} = -\alpha(\vec{n},\nabla T) = \frac{\alpha F(T_a - T(\vec{r}))}{\kappa}. \quad (8)$$

For preservation of parallelism of aggregate gradients it is necessary that $\dfrac{|\nabla\tilde{\phi}_1|}{\dfrac{\partial\tilde{\phi}}{\partial n}} = \dfrac{|\nabla T_1|}{\dfrac{\partial T}{\partial n}}$. That means $|\nabla\tilde{\phi}_1| = \alpha|\nabla T_1|$ while the relation between them is given by Eq. (4). Therefore, generally in case of heat exchange with the environment the resulting vectors $\nabla\tilde{\phi}_1$ and ∇T_1 do not remain parallel, which inevitably results in curl currents.

As an example we consider an element consisting of quasi-one-dimensional pellets (Fig. 3).

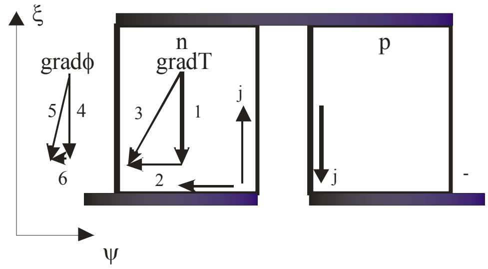

Figure 3. A schematic image of the thermal element with vectors of currents and gradients of temperature and potential.

Arrow j indicates the electric current direction. We assume that the current is directed so that the common junction of n- and p- type pellets should cool down. In quasi-one-dimensional case vector 1 equals temperature gradient $\kappa\frac{dT}{d\xi}$. In case of heat exchange with environment the direction of this vector changes to vector 2 that equals (8) $\frac{a(T_a - T(\vec{r}))}{\kappa}$. Vector 4 is the vector of the potential gradient, which in quasi-one-dimensional case is equal to $-\frac{j}{\sigma} + \alpha\frac{dT}{d\xi}$, where $j \neq 0$, and vector 6 equals $-\frac{a\alpha(T_a - T(\vec{r}))}{\kappa}$. Comparing triangles *grad T* and *grad*$\tilde{\phi}$, we see that they are not similar and vectors 3 and 5 are not collinear, therefore, close to the surface of the pellet in conditions of heat exchange with environment there are curl currents, which result in reduction of thermoelectric efficiency of the pellet.

Thus any inhomogeneity induced in quasi-one-dimensional pellets should result in curl currents, as they interfere with it being quasi-one-dimensional. The exception refers to the inhomogeneity along the basic spatial coordinate of the pellet that leaves the pellet quasi-one-dimentional.

Conclusions

Traditionally all divergences between the efficiency (Figure-of-Merit) of thermoelectric material Z and the efficiency of the thermoelectric module are explained by the contact electric resistance. Nevertheless, the direct Z measurements of materials based on bismuth chalcogenides frequently give values Z = 3.2 – 3.3 10^{-3} K^{-1} (sometimes even as high as 3.5 10^{-3} K^{-1}). The Figure-of-Merit, calculated considering temperature dependencies of thermoelectric parameters, for modules never gives anything similar. This can be caused not only by contact resistances but also by the occurrence of near-to-surface curl currents in a pellet. The measurements of material Z are commonly carried out on large-section samples (from tens to hundreds square millimeters). It is done to reduce the influence of the geometrical factor and most of modern miniature modules have pellets cross-section around a millimeter and less. In larger pellets the ratio of their perimeter to area is smaller thus decreasing the impact of curl currents. Unfortunately it is difficult to allocate this effect, as besides the influence of curl currents, with the decrease of the pellet section we get an increase the influence caused by broken surface layer due to pellet cutting.

Besides, in real modules for pellets on the edge on the module the thermal conditions on the outer surface of the pellet would be different from the thermal conditions on the inner surface of the pellet, which can result in stronger distortions of temperature field compared to adiabatically isolated pellet. Thus the efficiency of the periphery pellets can decrease more that the efficiency of the pellets inside the module, which also should result in reduction of the module Z-value compared to efficiency of the thermoelectric material.

References

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